Monochromatic cycle partitions

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University of Cambridge

SIAM Conference on Discrete Mathematics
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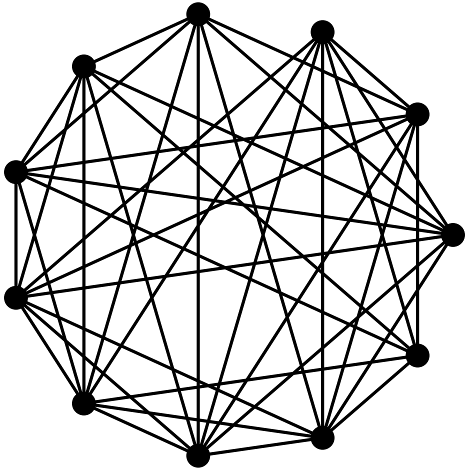

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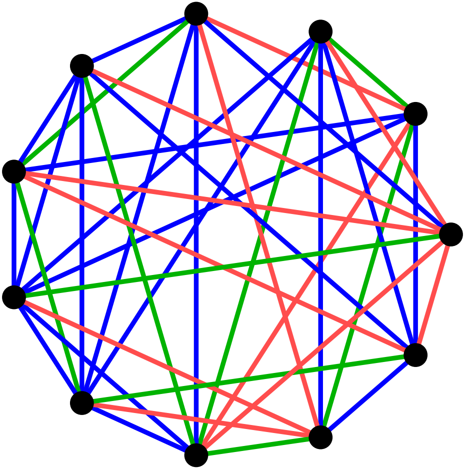

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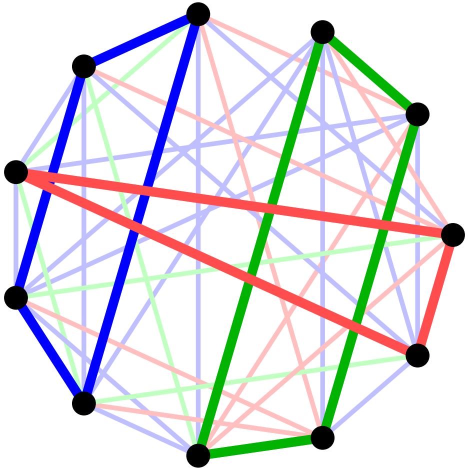

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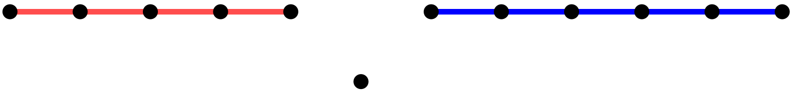

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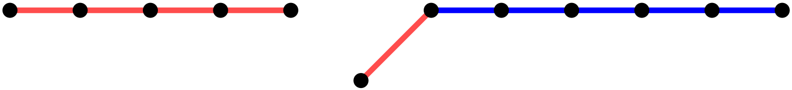

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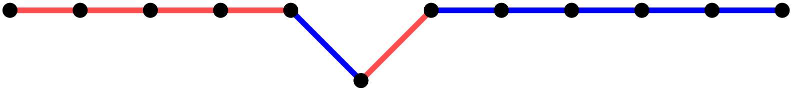

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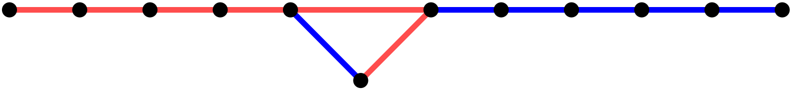

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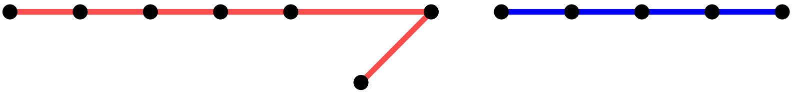

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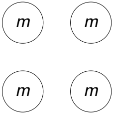

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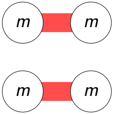

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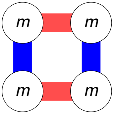

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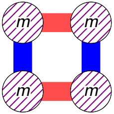

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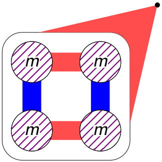

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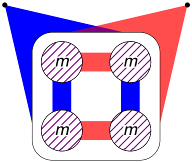

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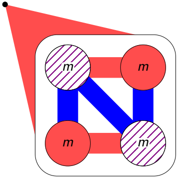

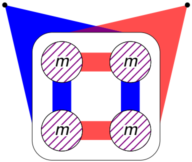

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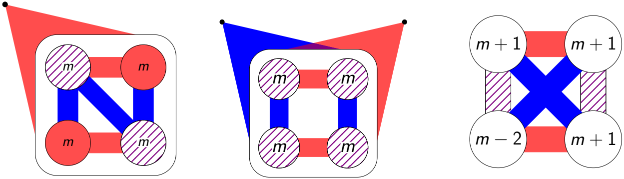

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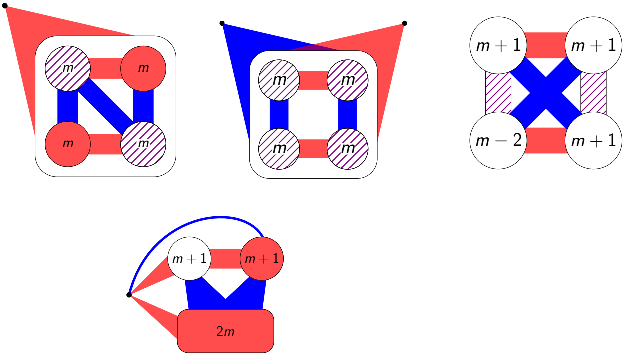

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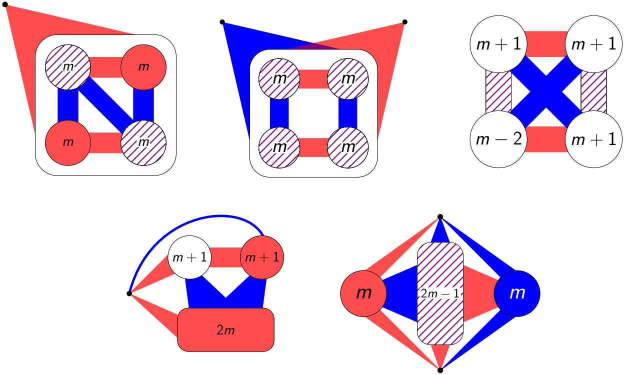

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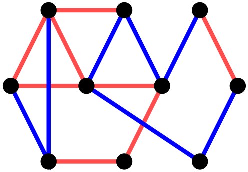

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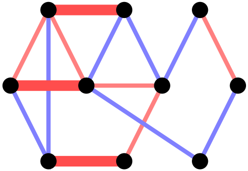

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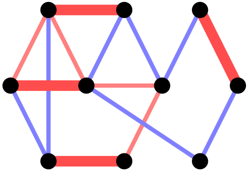

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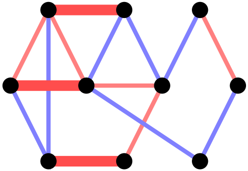

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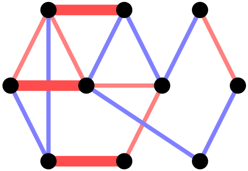

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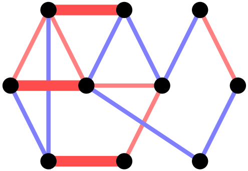

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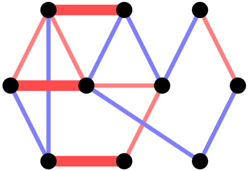

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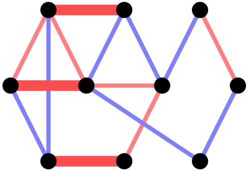

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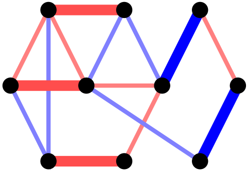

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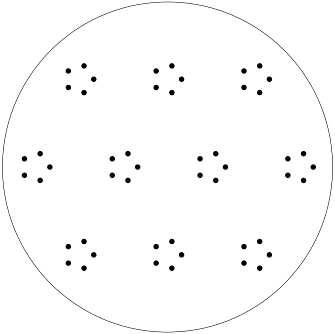

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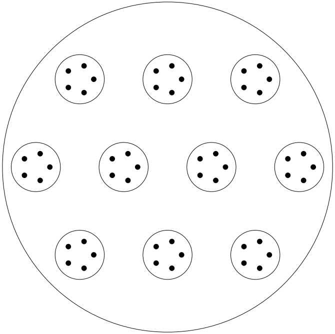

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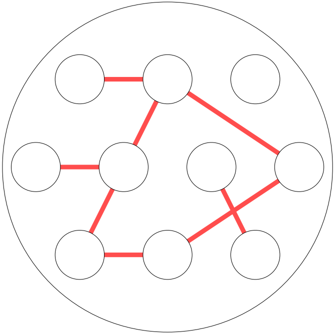

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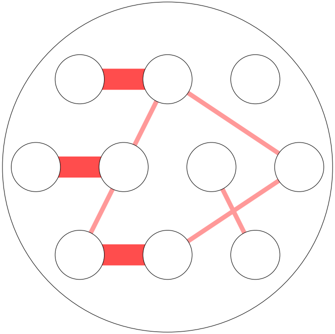

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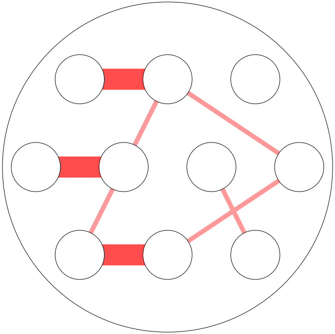

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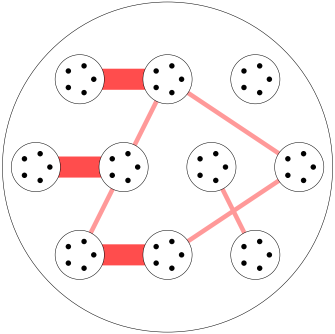

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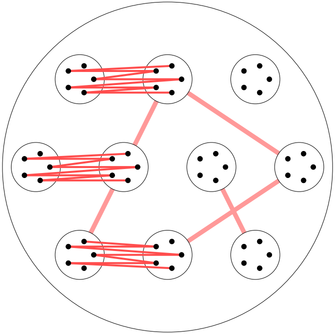

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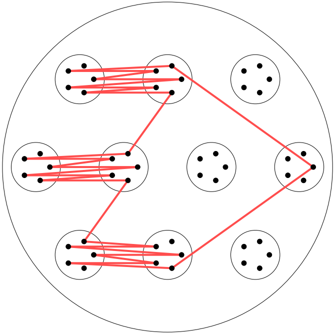

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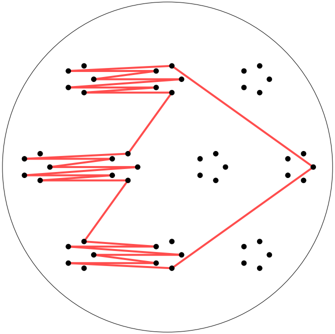

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Sketch of proof: first approximate result

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- Find the required cycles.

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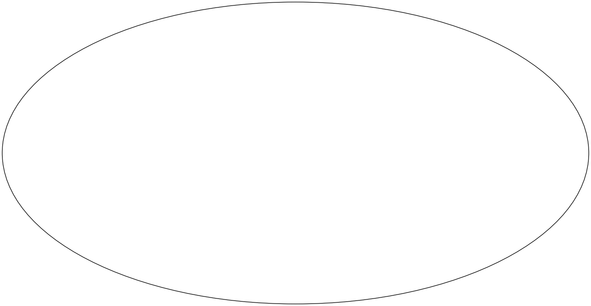

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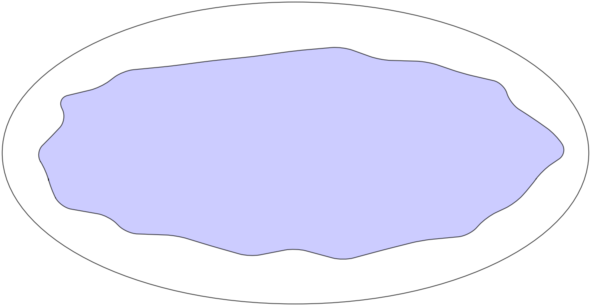

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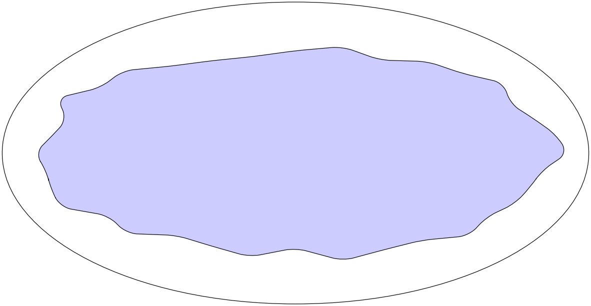

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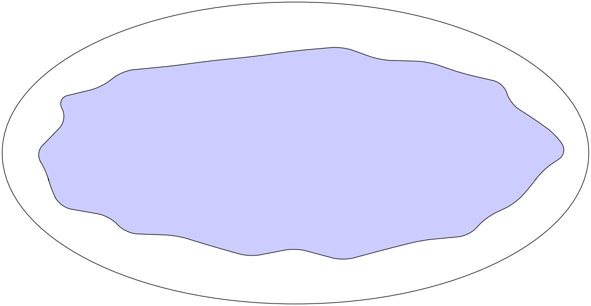

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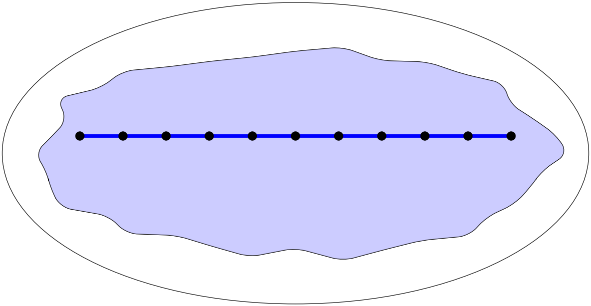

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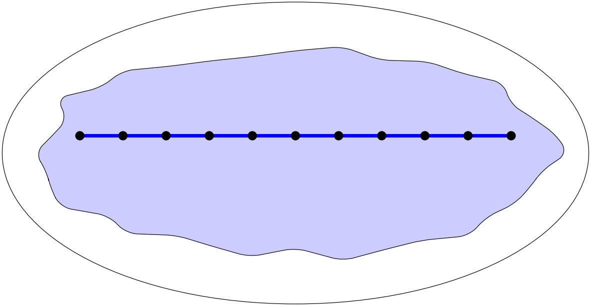

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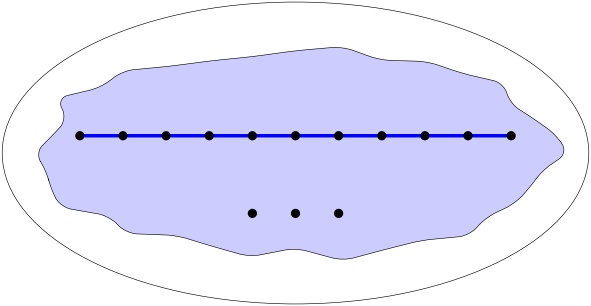

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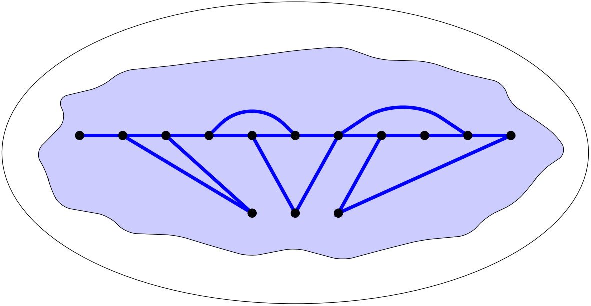

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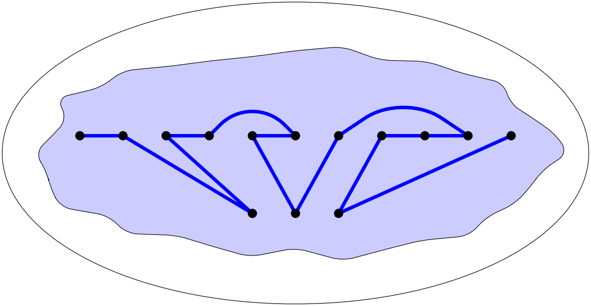

Sketch of second approximate result

Proof of DeBiasio and Nelsen's theorem:

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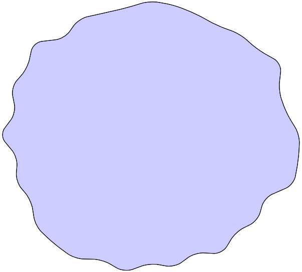

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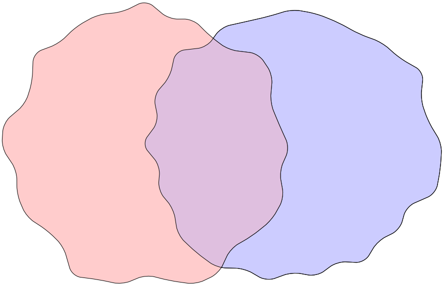

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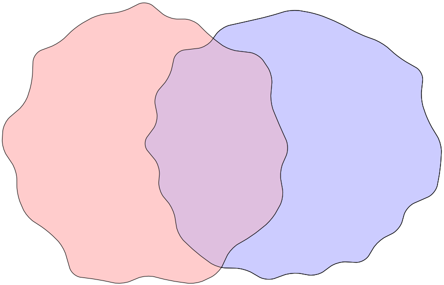

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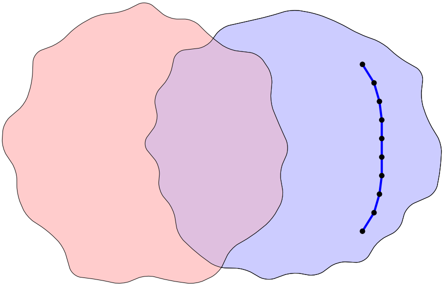

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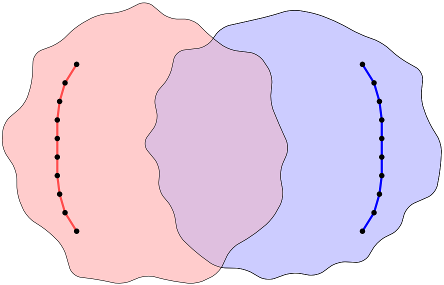

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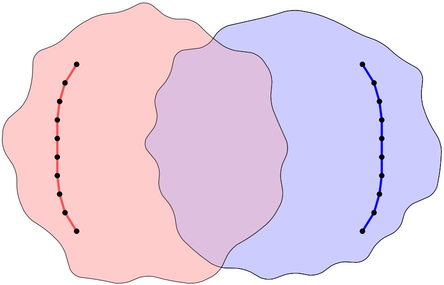

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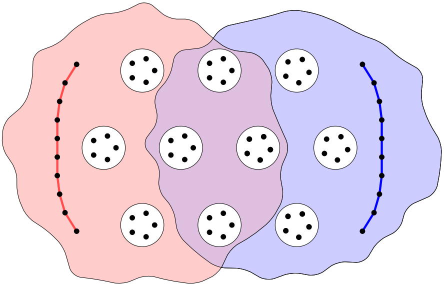

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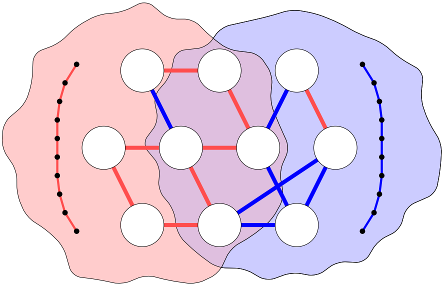

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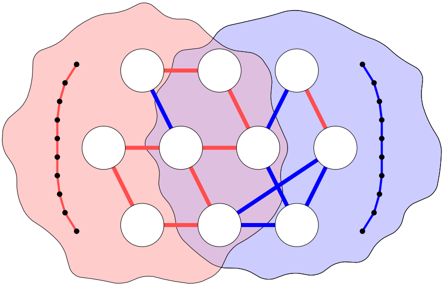

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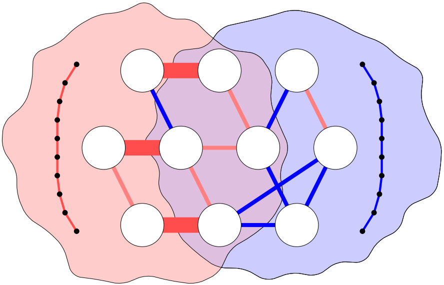

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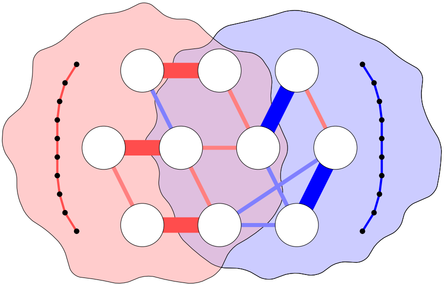

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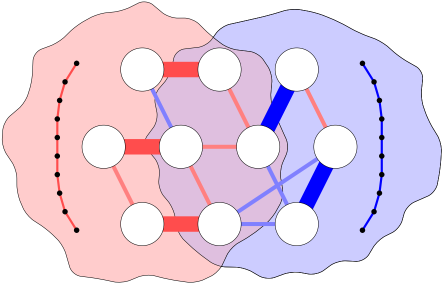

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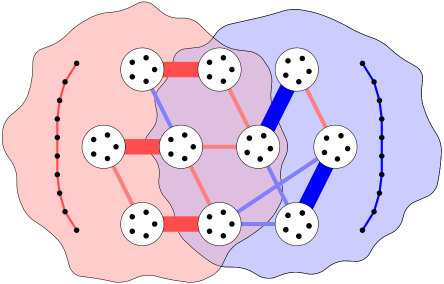

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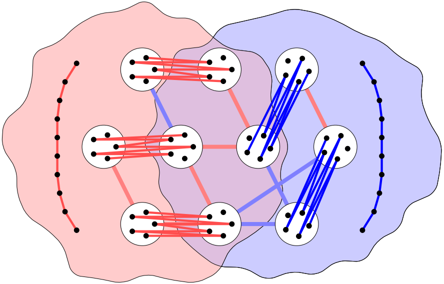

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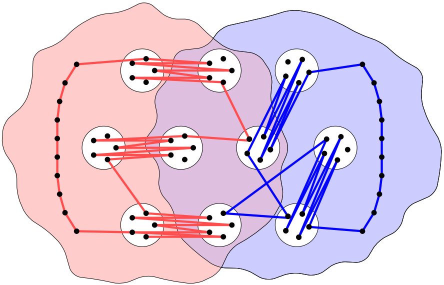

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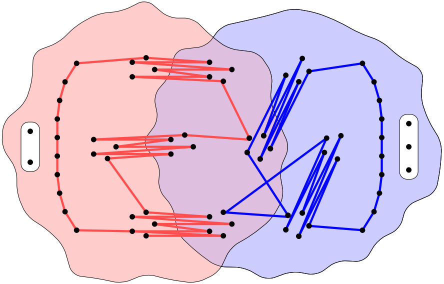

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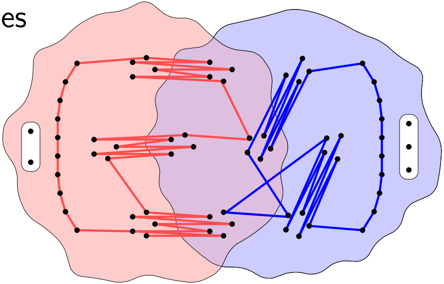

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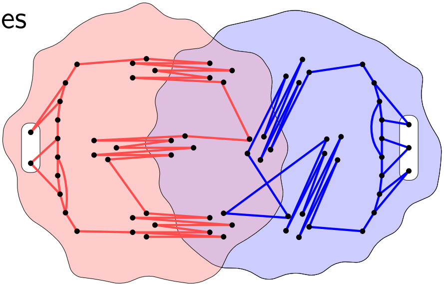

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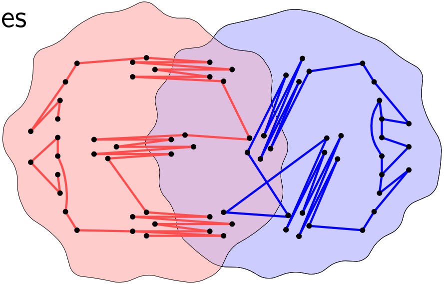

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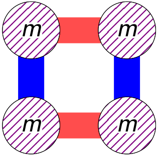

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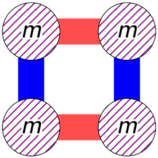

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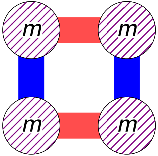

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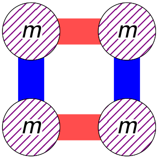

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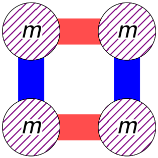

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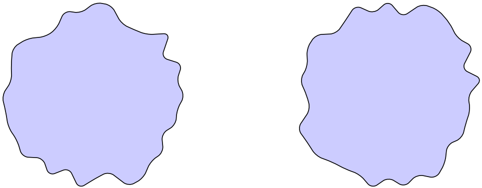

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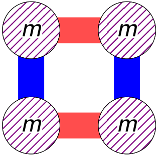

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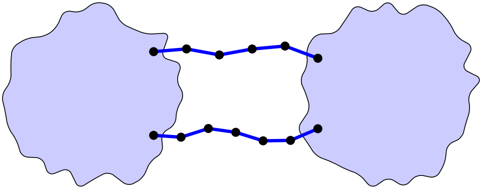

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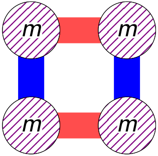

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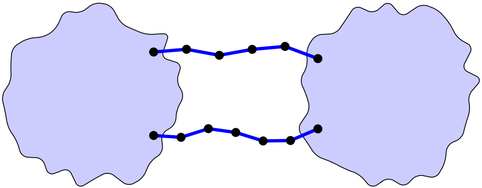

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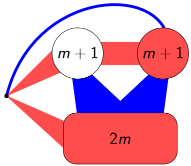

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May have $\delta(\Gamma) < 3|\Gamma|/4$.
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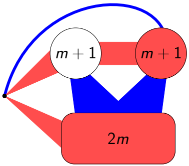

The end

Thank you for listening!