Finding monotone patterns

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ETH–ITS

joint with Omri Ben-Eliezer, Clément Canonne, Erik Waingarten

Mittagsseminar

December 2019

Property testing

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We consider testing with **one-sided error**: if an object is far from having $\mathcal{P}$, provide evidence.

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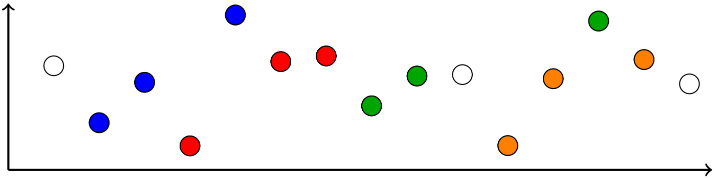
Fix $k \geq 2$.

- **Input.** $f : [n] \to \mathbb{R}$ with $\Omega(n)$ disjoint increasing k-tuples.



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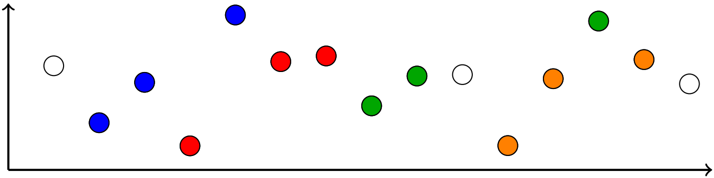

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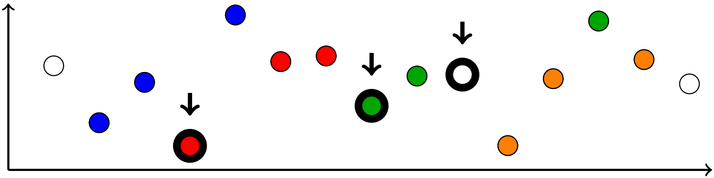

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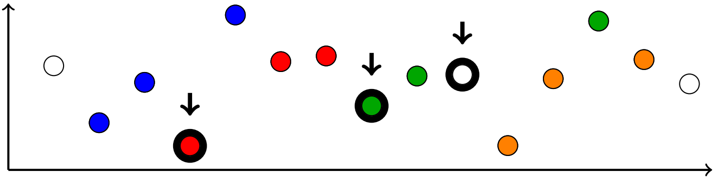

We sometimes refer to an increasing k-tuple as a (**1**...**k**)-**copy**.

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Newman, Rabinovich, Rajendraprasad, Sohler '17. For $k \geq 2$, there is a (non-adaptive) tester which makes $(\log n)^{O(k^2)}$ queries.

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Theorem (Ben-Eliezer, Canonne, L., Waingarten)

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Theorem (Ben-Eliezer, L., Waingarten)

An optimal adaptive algorithm for testing $(1\ldots k)$-freeness makes $\Theta_k(\log n)$ queries.

Towards a lower bound for $k = 2$: binary profiles

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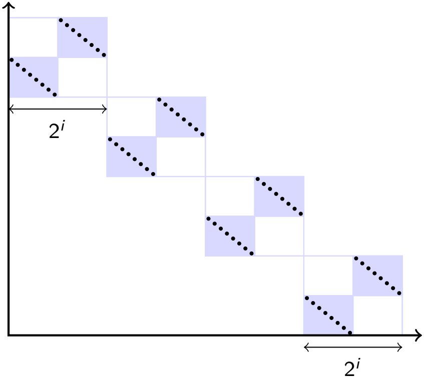

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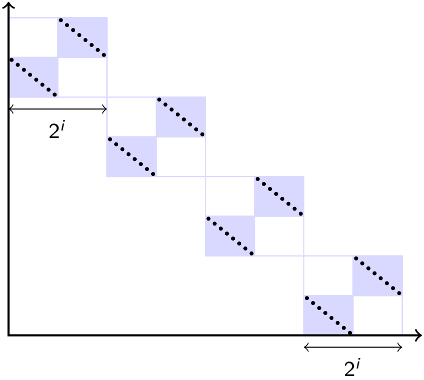

Observation

If (x, y) is an increasing pair in f_i, then $b(x, y) = i$.

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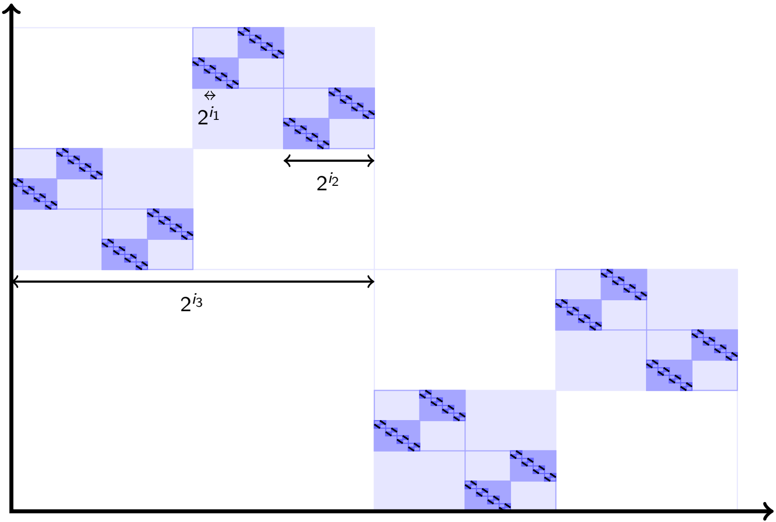
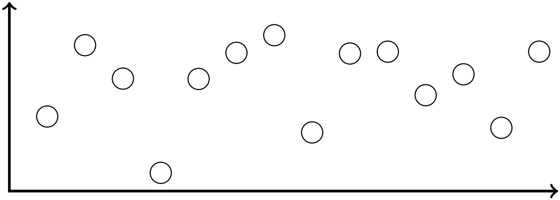
Thus, to find an increasing pair with probability at least 0.99 need at least $0.99 \log n$ queries.

Lower bound for $k = 2^\kappa$: iterated construction



Choosing copies greedily



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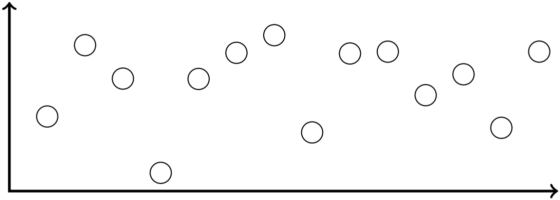

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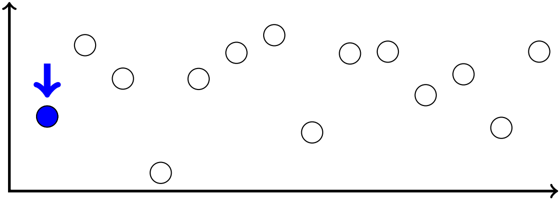

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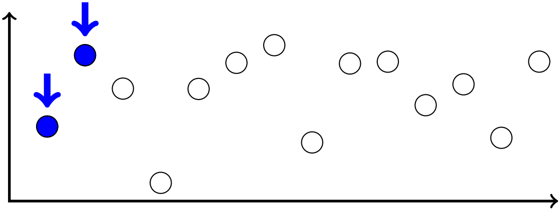

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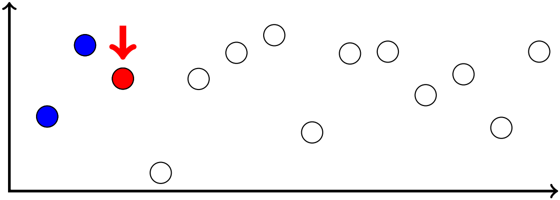

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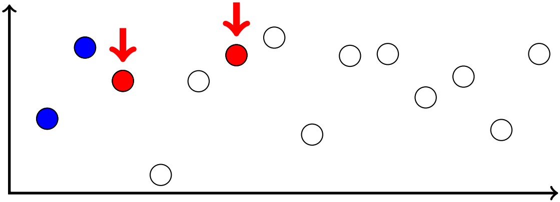

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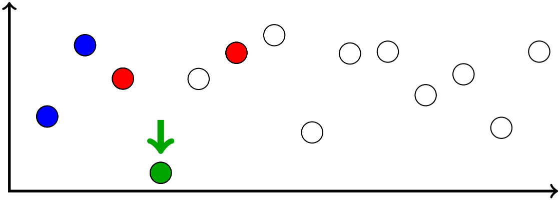

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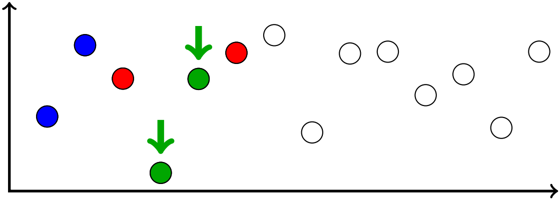

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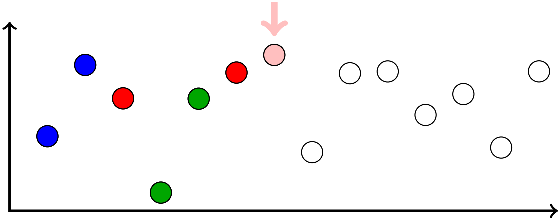

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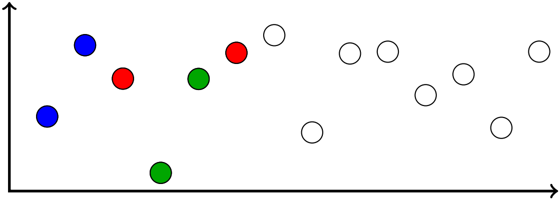

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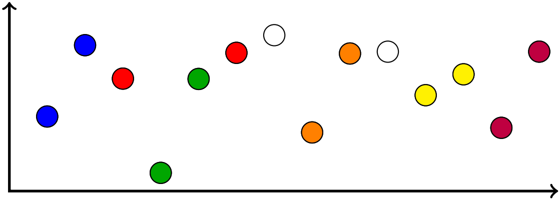

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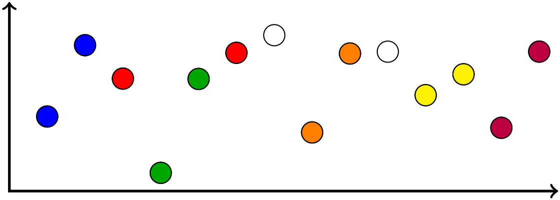

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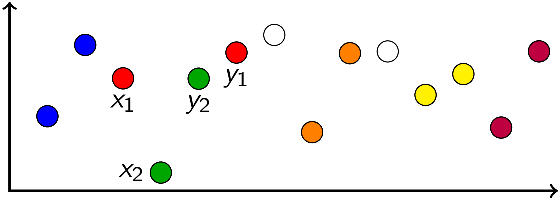
Choosing copies greedily



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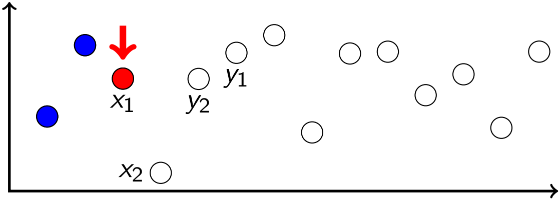

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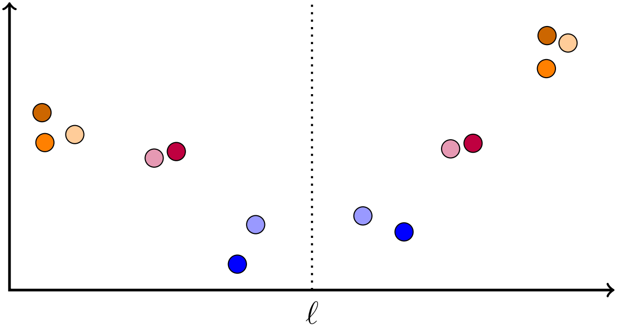

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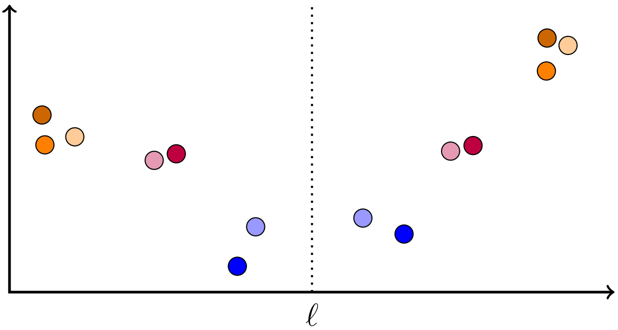

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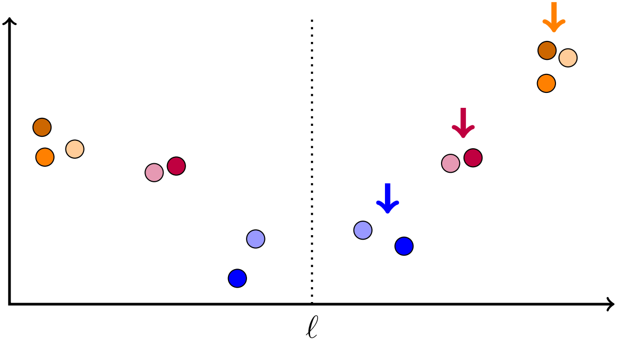

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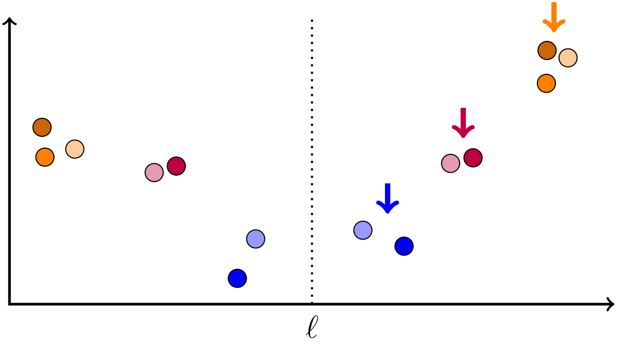

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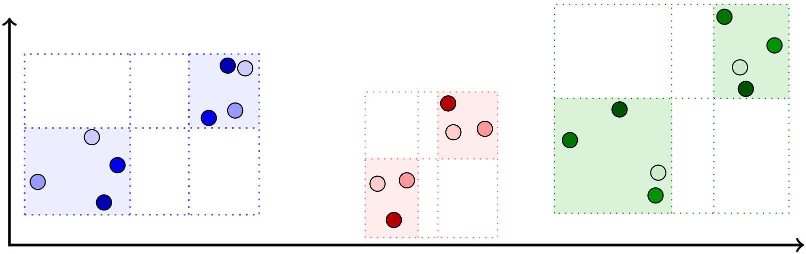

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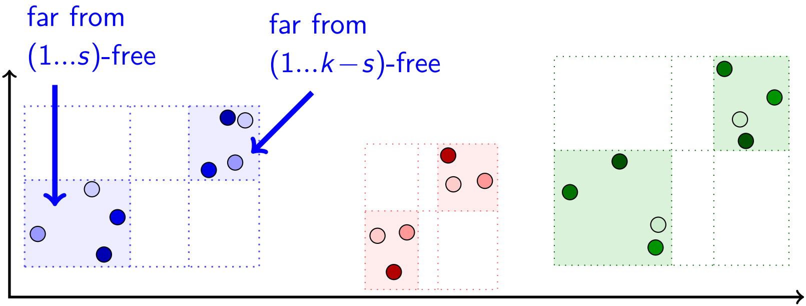

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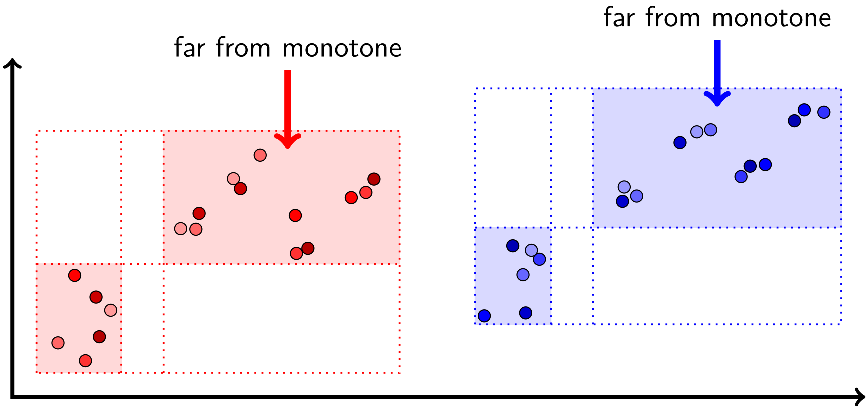

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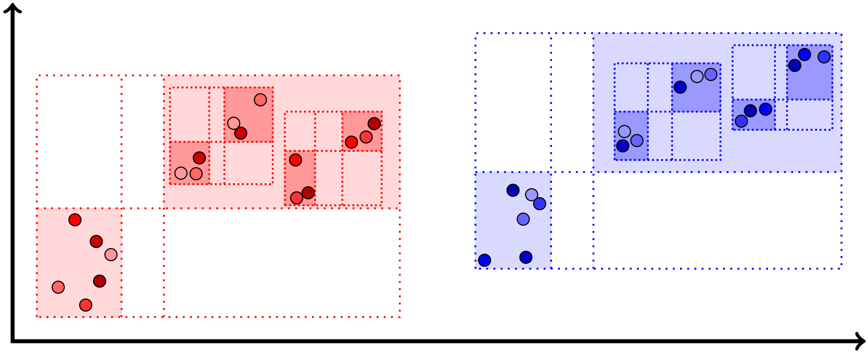

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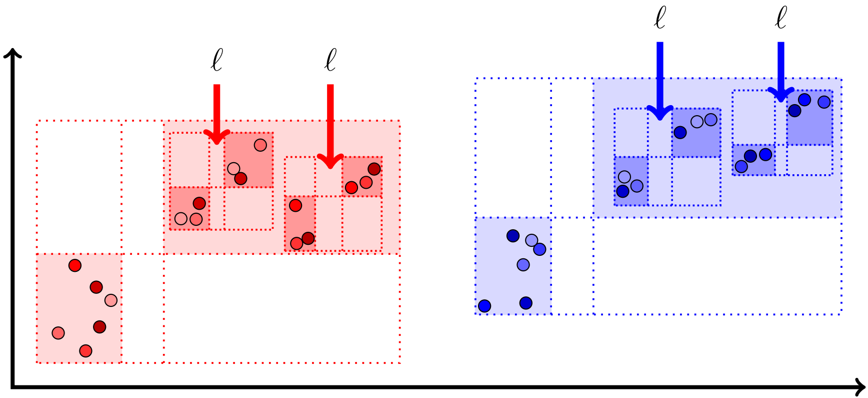

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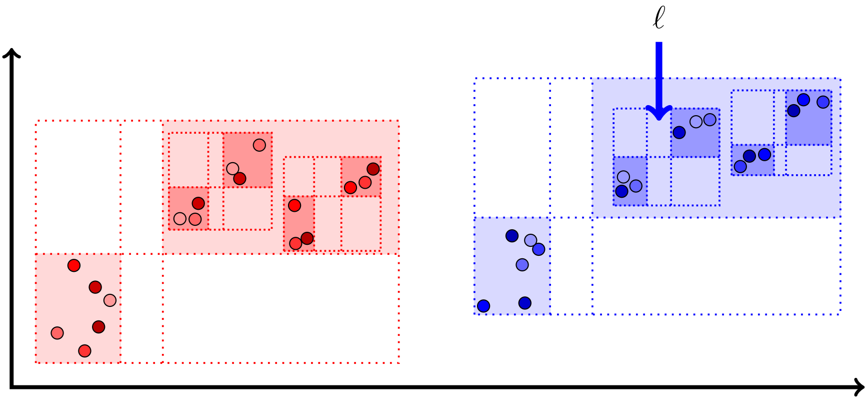

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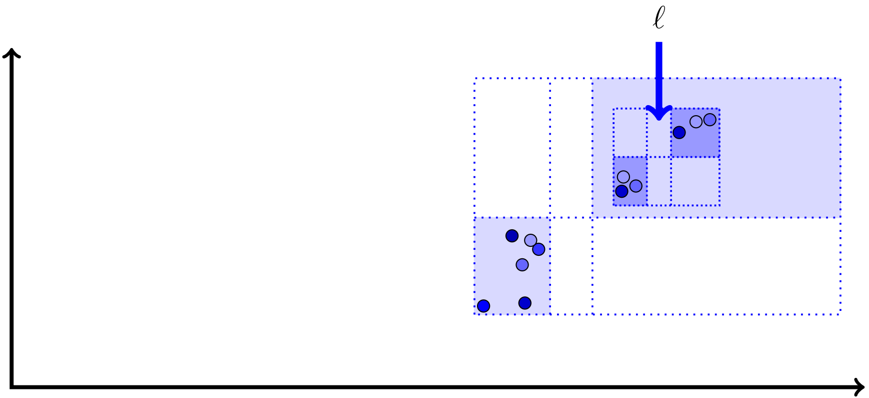

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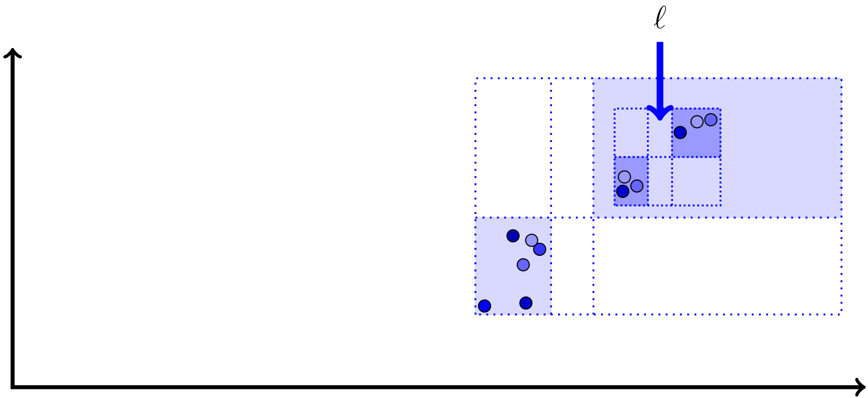

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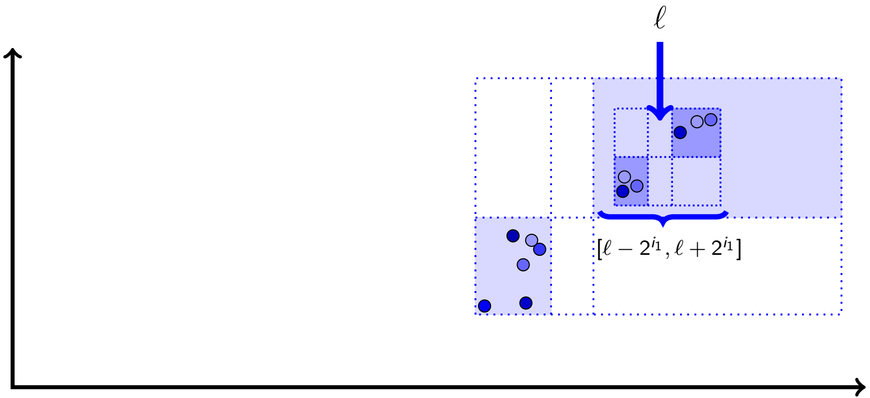

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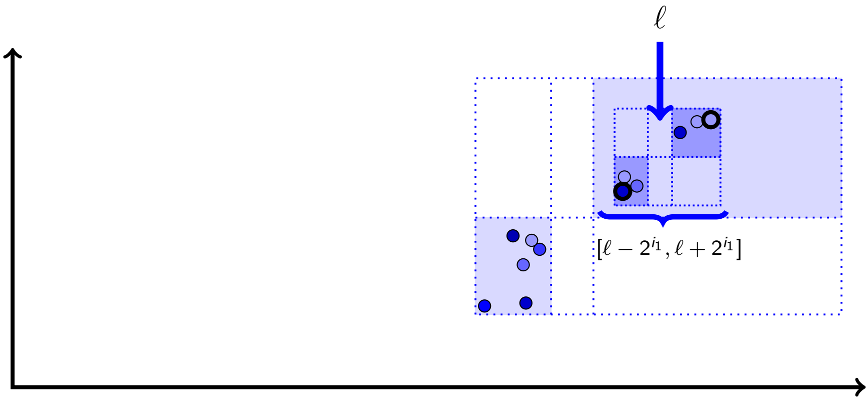

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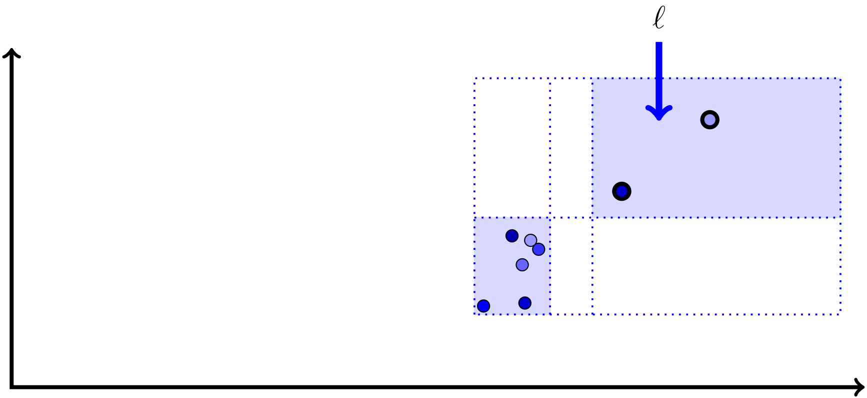

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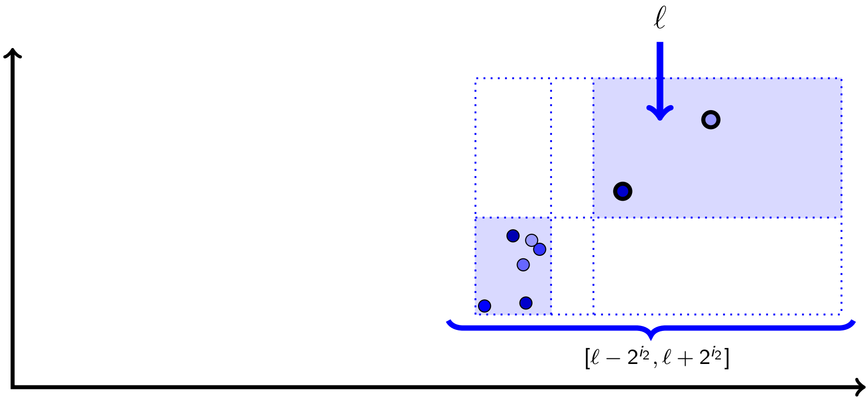

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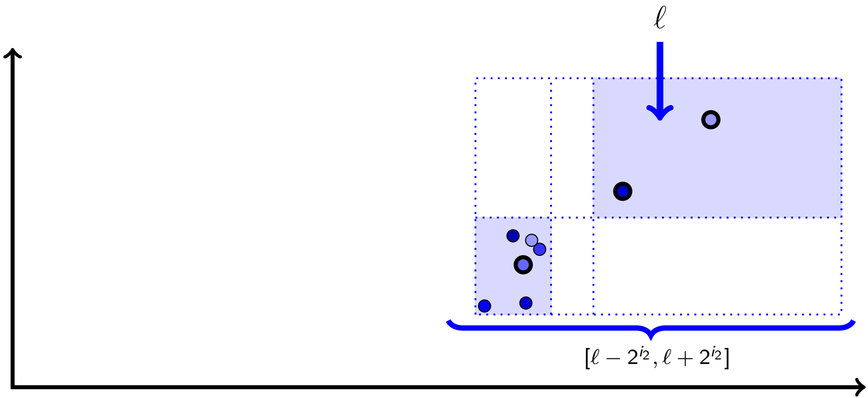

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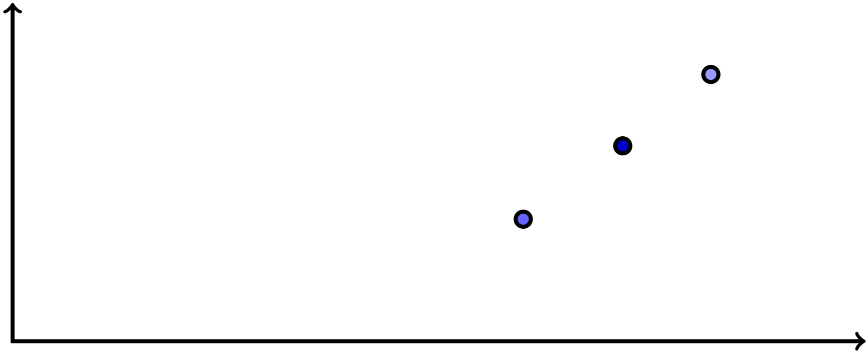

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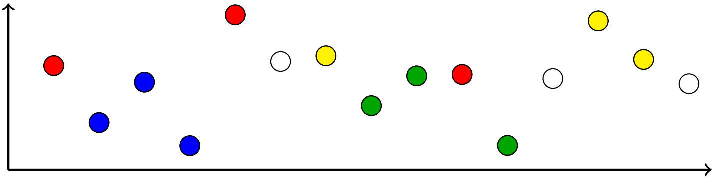

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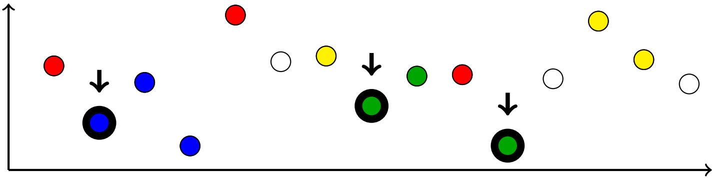

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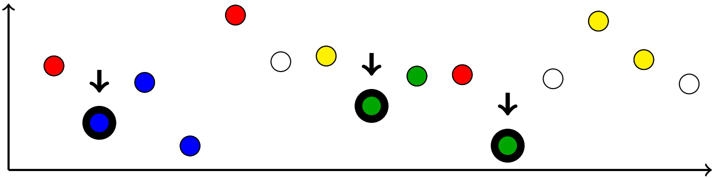

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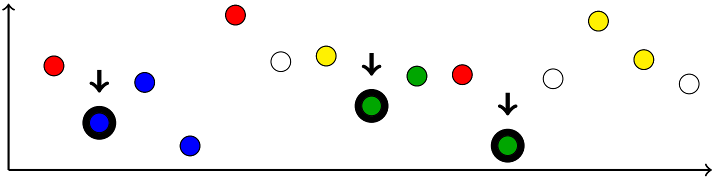

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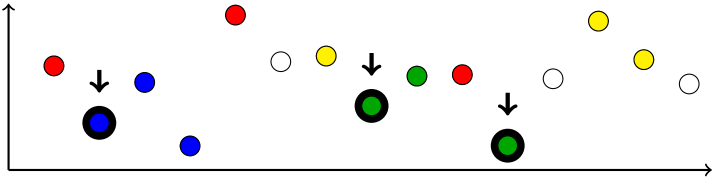

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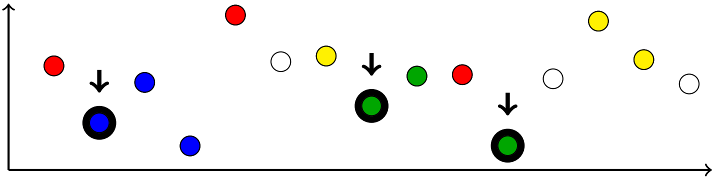

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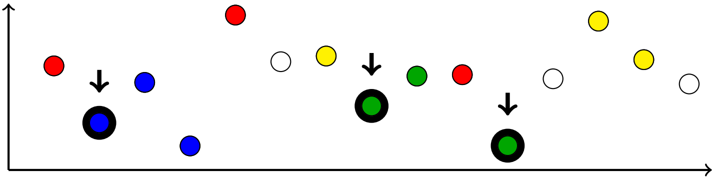

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