Path partitions of regular graphs

Shoham Letzter

joint work with Vytautas Gruslys

University of Cambridge and ETH–ITS

SIAM DM

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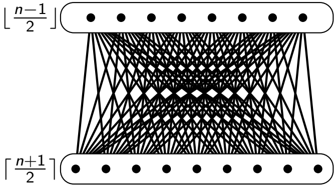

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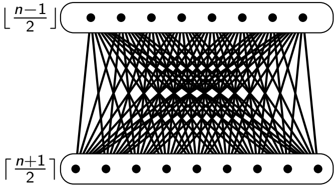

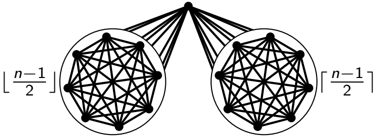

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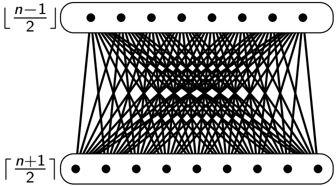

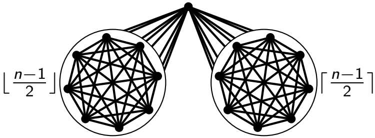

Answer. Need to circumvent both examples!

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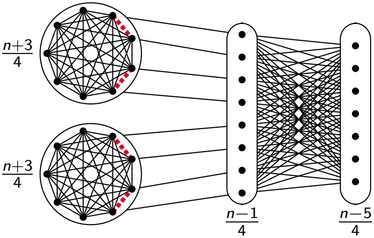

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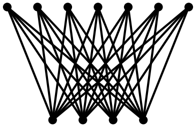

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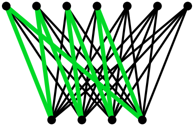

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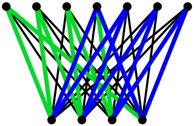

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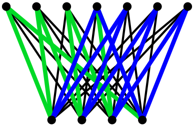

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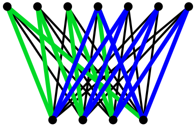

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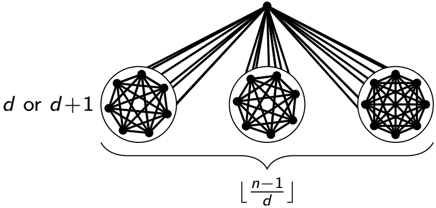

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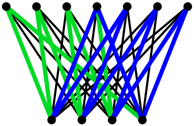

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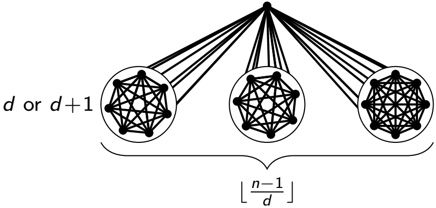

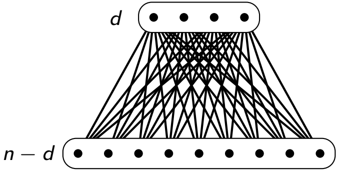

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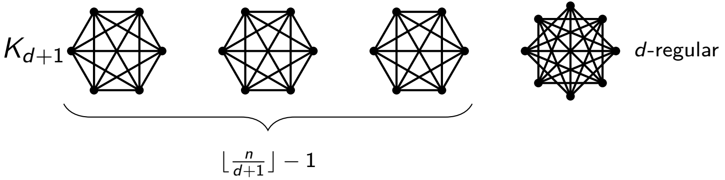

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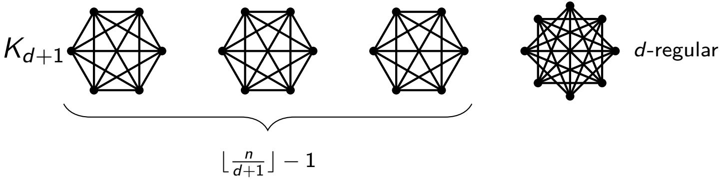

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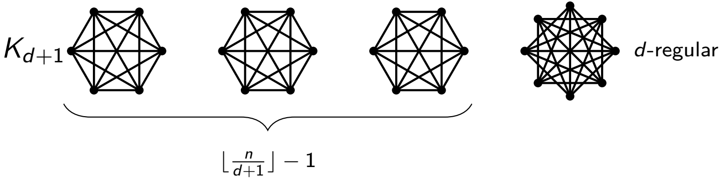

- **Magnant, Martin ('09).** $d \leq 5$.
- **Han ('17).** If $d \geq cn$ then all but $o(n)$ vertices can be covered by $\lfloor \frac{n}{d+1} \rfloor$ vertex-disjoint paths.

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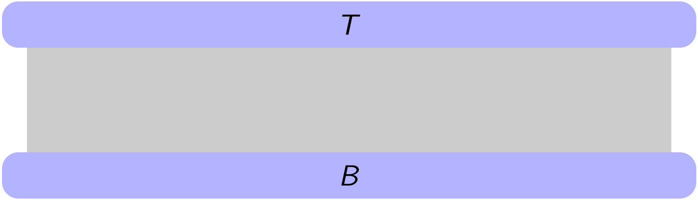
- Standard applications use Regularity Lemma. Here it is possible to avoid it, using an argument by Lo and Patel ('15) which uses the 'Rotation-Extension' technique.

Partition vertices into clusters



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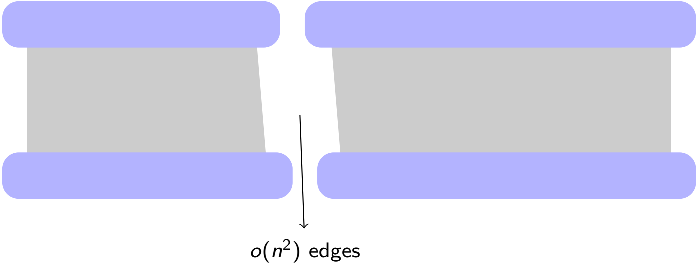

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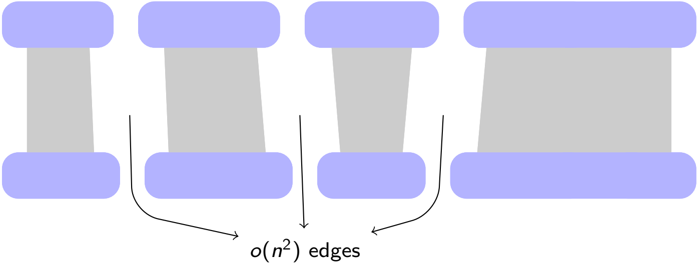

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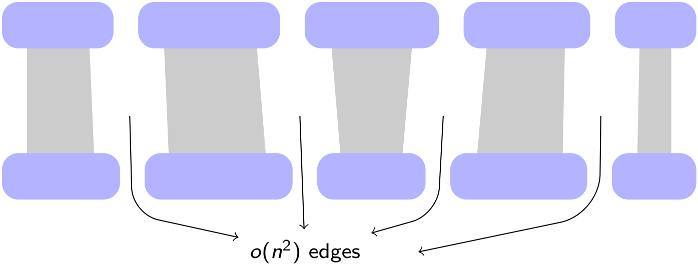

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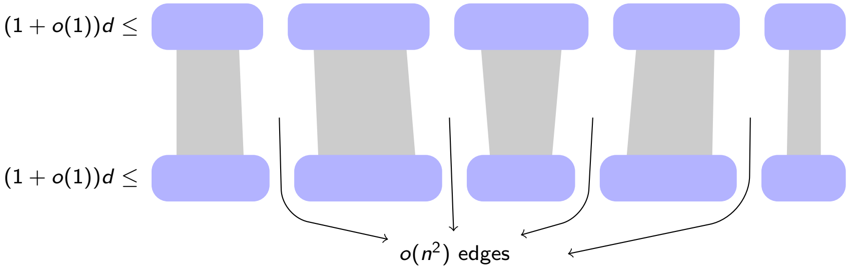

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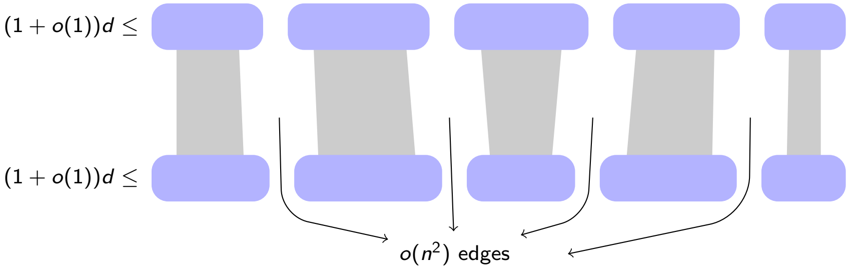

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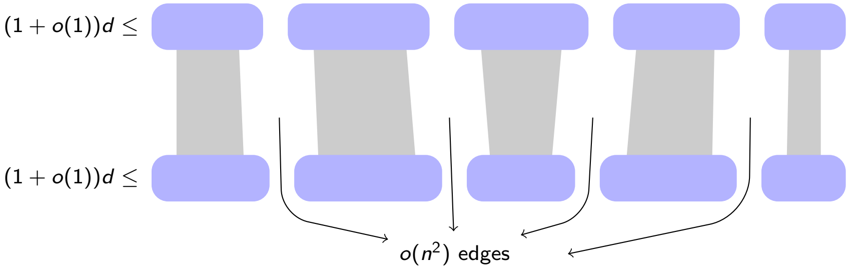

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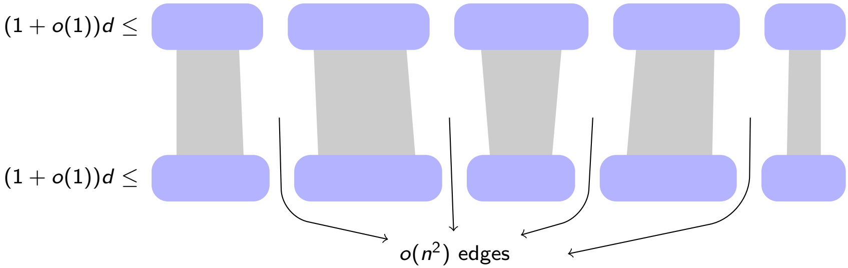

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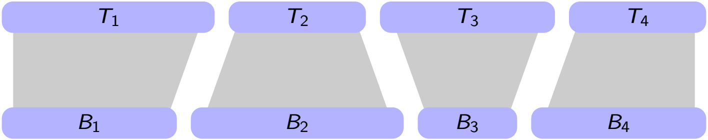

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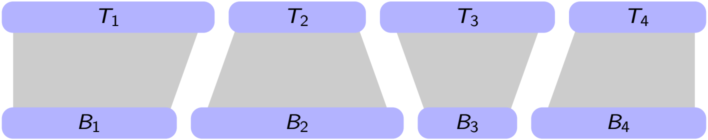

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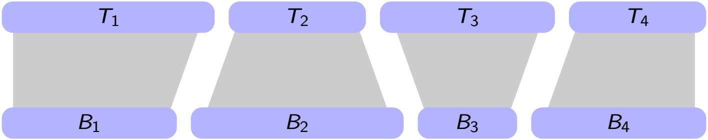

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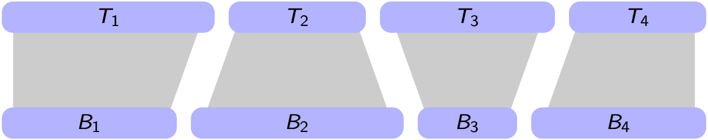

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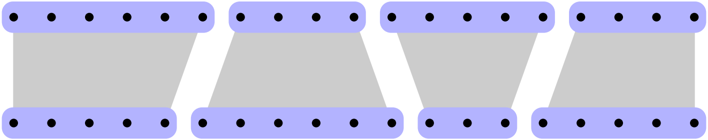

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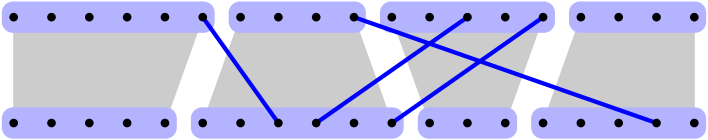

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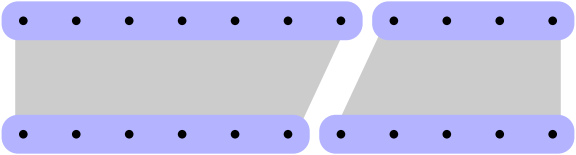

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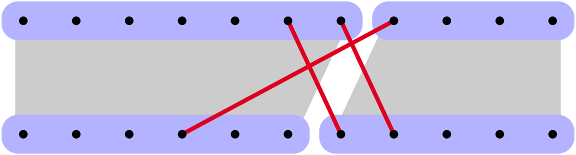

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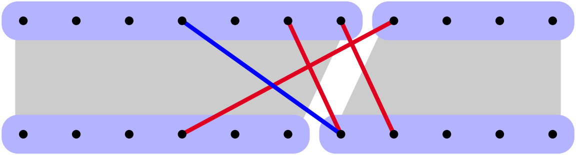

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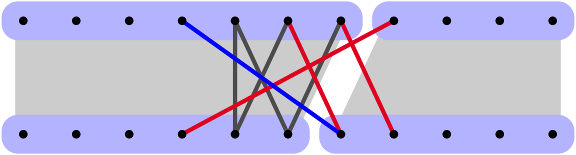

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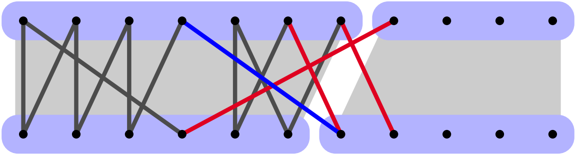

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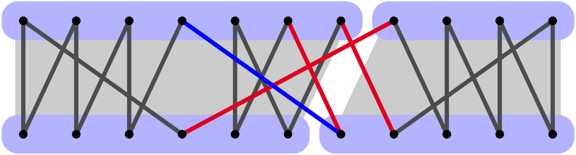

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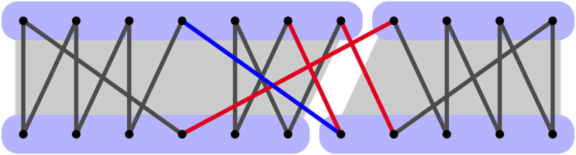

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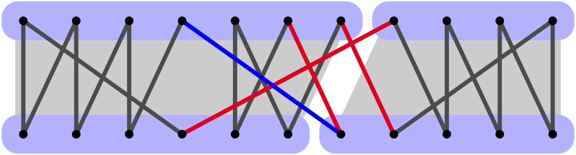

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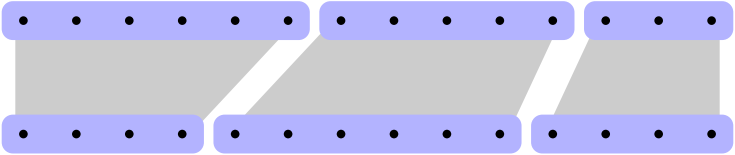

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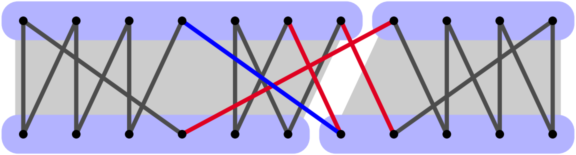

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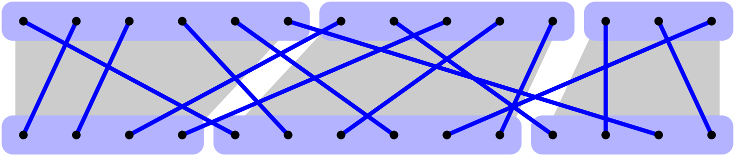

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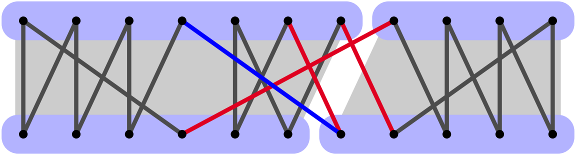

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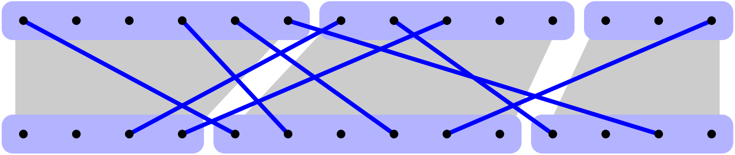

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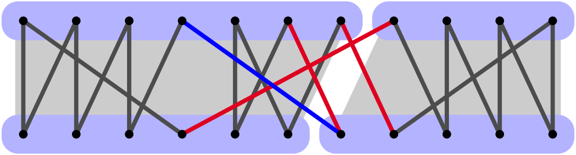

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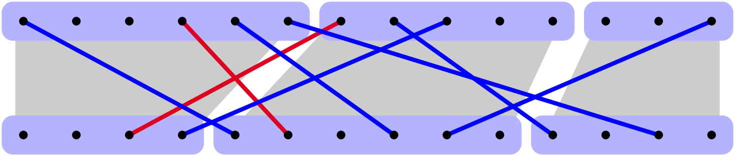

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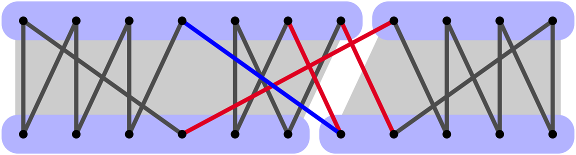

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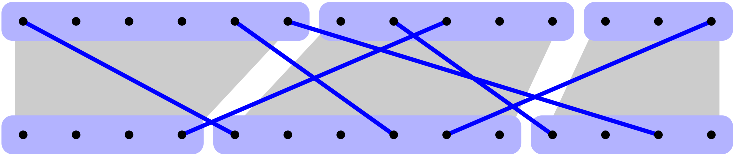

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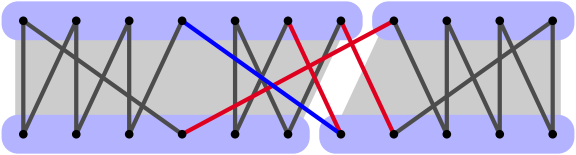

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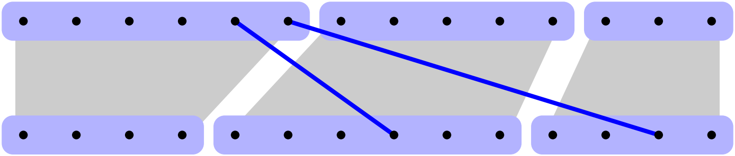

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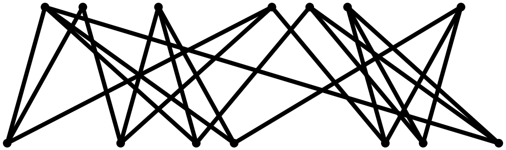

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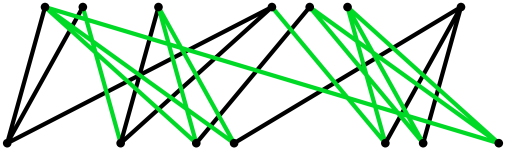

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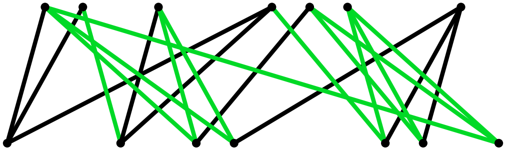

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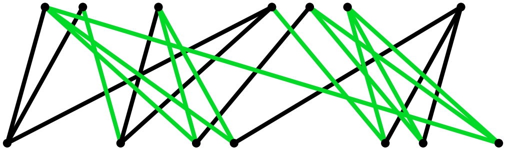

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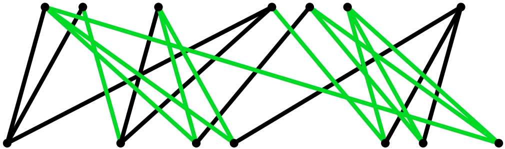

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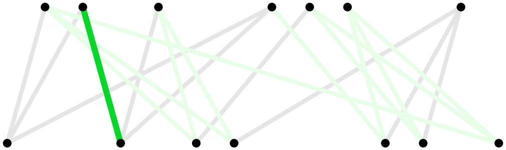

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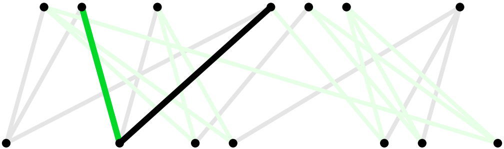

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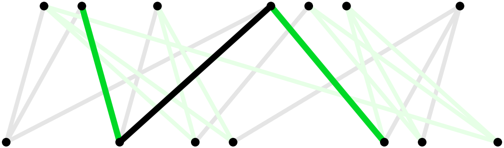

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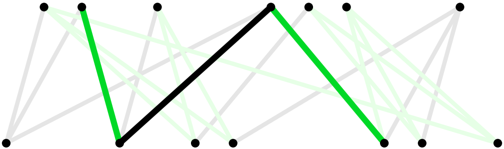

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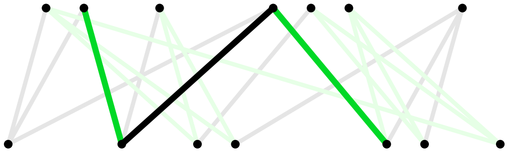

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Suffices to show. If σ is random, G_σ has a balancing matching with probability larger than $1/2$.

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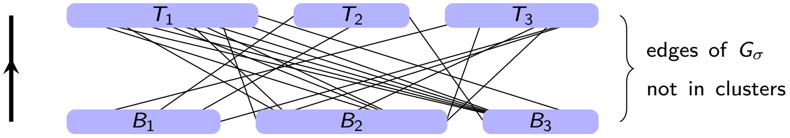

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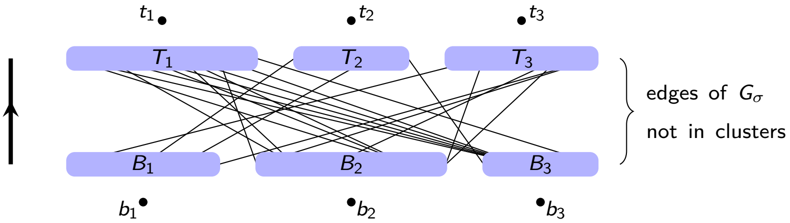

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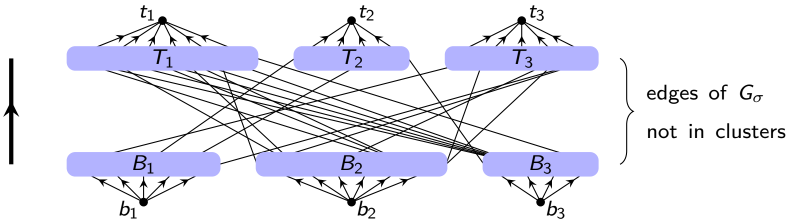

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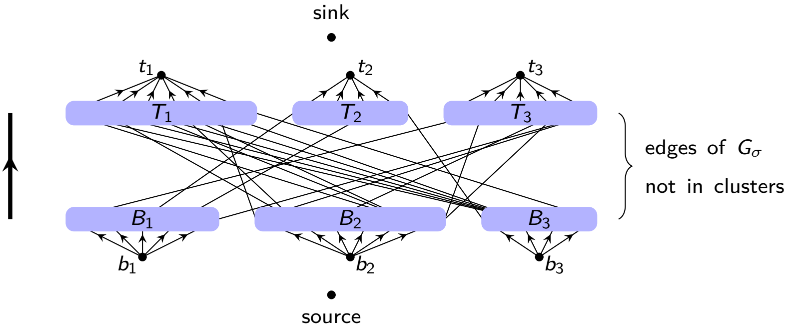

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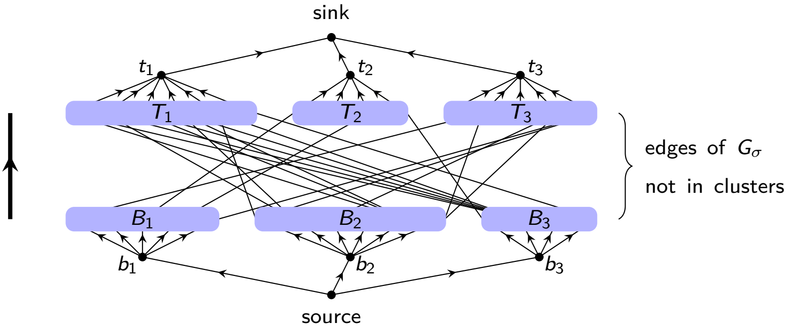

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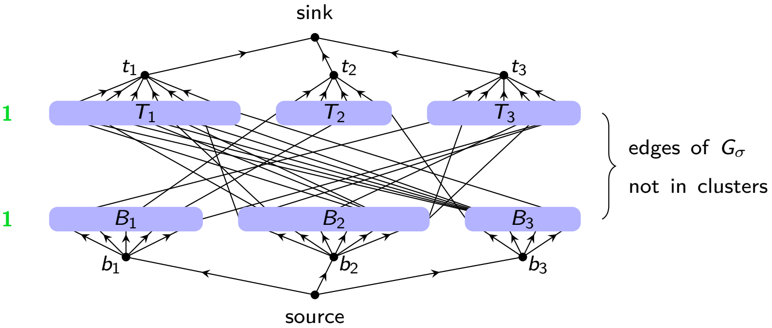

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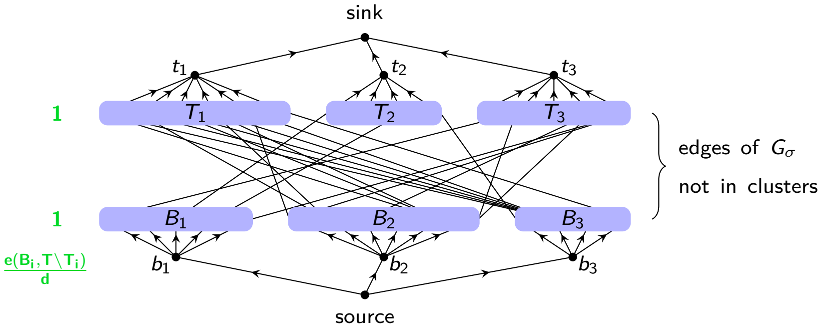

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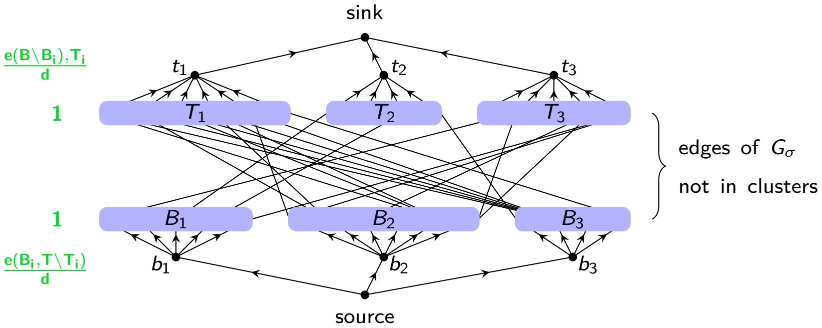

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With probability $> \frac{1}{2}$, *there is a flow with value* $\sum_{i \neq j} \frac{e(B_i, T_j)}{d} - \frac{9}{10}$.

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2-lifts

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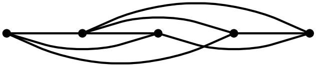

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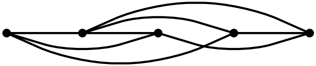

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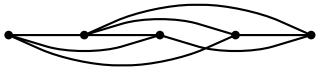

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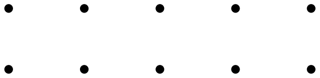

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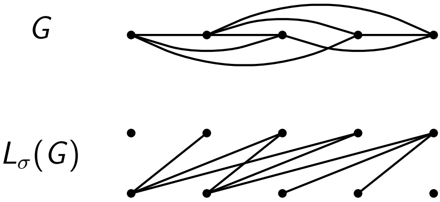

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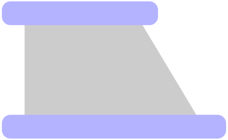

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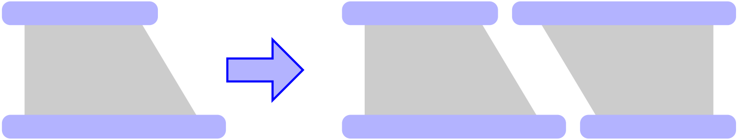

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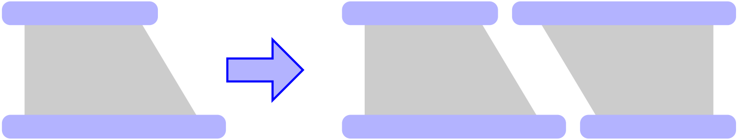

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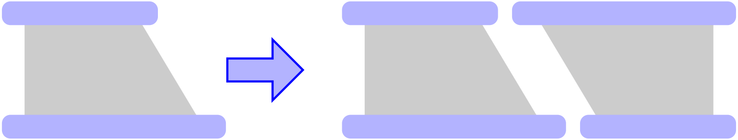

- far-from-bipartite clusters become one balanced cluster,
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Theorem (Gruslys, L. '17+)

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